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Free convection from a semi-infinite vertical surface bounded by a horizontal wall in a porous medium

D. B. INGHAM
Department of Applied Mathematical Studies, University of Leeds, Leeds LS2 9JT, U.K.

and

I. POP
Faculty of Mathematics, University of Cluj, R-3400, Cluj, Romania

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Abstract—A study of the steady free convection flow along a semi-infinite vertical flat plate which is placed in a saturated porous medium and at an arbitrary distance d above a horizontal wall is performed. The first- and second-order boundary layer equations and outer inviscid flow equations are studied to find the effects of large, but finite, values of the Rayleigh number. Results are obtained for the two cases in which the flat plate is (i) isothermal and (ii) uniform flux. It is found that the first-order boundary layer solutions overestimate the local Nusselt number except for the uniform flux plate for distances along the plate less than about $0.4d$.

1. INTRODUCTION

PROBLEMS of natural convection in porous media have become of considerable interest during the last two decades. This has been mainly due to its wide engineering applications such as geothermal energy resource and oil-reservoir modelling and in the analysis of insulating systems. An excellent review of these types of problems can be found in Bejan [1].

Much work in this area has been devoted to the analytical studies of higher-order boundary layer effects from heated surfaces based on the method of matched asymptotic expansion. Cheng and Hsu [2] and Joshi and Gebhart [3] have obtained higher-order corrections for the free convection boundary layer flow adjacent to a semi-infinite vertical flat plate which is embedded in a saturated porous medium. This flow configuration has been recently extended by Daniels and Simkins [4] who investigated the flow in a corner which is formed by a uniformly heated vertical surface and a second, thermally insulated wall which meets the vertical wall at the origin. Further Hsu and Cheng [5] have considered free convection about a semi-infinite inclined heated surface which intercepts with another unheated surface embedded in a porous medium. The same method was applied by Riley and Rees [6] to the problem of non-Darcy natural convection from an arbitrarily inclined heated surface in a porous medium.

The problem considered here is the steady free convection flow along a semi-infinite vertical flat plate which is placed at a distance d above an insulated horizontal infinite wall which is immersed in a saturated porous medium. In spite of its relevance to many practical situations, this configuration has not been previously analysed. The only related work to this problem is that performed by Martynenko *et al.* [7] who investigated the situation of a viscous fluid. We shall consider two natural convection flows separately, namely, (i) an isothermal flat plate and (ii) a uniform flux flat plate. Both cases are analysed by the method of matched asymptotic expansions. The perturbation parameter is the inverse of the square root of the Rayleigh number which is assumed to be large in order to ensure the existence of the free convection boundary layer. It is found that the second-order boundary layer equations reduce to a set of non-similar equations. These equations have been solved numerically for small and large values of the coordinate along the plate in order to provide descriptions of the velocity and temperature fields. These limiting solutions are matched using a numerical procedure to solve the full governing parabolic partial differential equations. It was noted that the outer flow and the boundary layer flow patterns are considerably different in the two problems considered in this paper. This work is analogous to the viscous flow induced by a horizontal line source of heat which is bounded by a horizontal infinite wall as studied by Riley [8] and Afzal [9].

2. PHYSICAL MODEL AND ANALYSIS

The configuration considered is that of a semi-infinite vertical flat plate which is embedded in a saturated porous medium and placed at a distance d above an infinite horizontal wall (see Fig. 1). The vertical plate is kept at a constant temperature, T_w, which is higher

NOMENCLATURE

d	shortest distance from the plate to the horizontal wall
g	acceleration due to gravity
k	thermal conductivity of the porous medium
K	permeability of the porous medium
Nu_x	local Nusselt number
q_w	local heat transfer rate
Ra_x	local Rayleigh number
T_w, T_0	surface and ambient temperatures
(x, y)	dimensionless coordinates
Y	dimensionless inner coordinate.

Greek symbols

α	equivalent thermal diffusivity
β	coefficient of thermal expansion
ε	perturbation parameter
η	similarity variable
θ	dimensionless temperature
ν	kinematic viscosity
ψ	dimensionless stream function
$\tilde{\psi}$	dimensionless outer stream function.

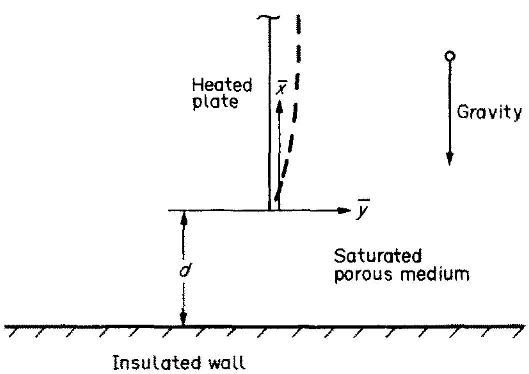

FIG. 1. Physical model and coordinate system.

than that of the temperature of the ambient fluid-porous medium, T_0, or at a constant flux, q_w, whilst the horizontal wall is maintained at the constant temperature T_0.

Cartesian coordinates $(\bar{x}, \bar{y})$ are used in the subsequent analysis in which $\bar{x}$ and $\bar{y}$ are measured along and perpendicular to the plate, respectively. The origin of the coordinate system is taken to coincide with the leading edge of the plate. We assume that the thermal and fluid-porous medium properties to be constant and neglect the viscous dissipation. Under steady-state conditions the mass, momentum and energy equations, with the Darcy–Boussinesq approximation describing the natural convection flow along the vertical plate can easily be obtained. These equations for the stream function ψ and the temperature θ can be written in non-dimensional form as [2]

$$\psi_{xx} + \psi_{yy} = \theta_y \tag{1}$$

$$\varepsilon^2(\theta_{xx} + \theta_{yy}) = \theta_x \psi_y - \theta_y \psi_x \tag{2}$$

where $\varepsilon = Ra^{-1/2}$ and Ra denotes the Rayleigh number which is based on the distance d, $Ra = g\beta K \Delta T d/(\alpha\nu)$. In equations (1) and (2) the streamfunction is non-dimensionalized by $g\beta K \Delta T d/\nu$, the lengths by d and the temperature by ΔT where

$$\Delta T = \begin{cases} T_w - T_0, & \text{for the isothermal plate} \\ q_w d/(k\,Ra^{1/2}), & \text{for the uniform flux plate.} \end{cases} \tag{3}$$

The associated boundary conditions of the plate described above take the following form:

$$\left.\begin{array}{l} \psi_x = 0,\ \theta = 1 \text{ (constant temperature)} \\ \text{or} \\ \varepsilon\theta_y = -1 \text{ (constant heat flux)} \end{array}\right\} \text{at } y = 0,\ x \geqslant 0 \tag{4a}$$

$$\psi_y \to 0,\ \theta \to 0 \qquad \text{as } y \to \infty,\ x \geqslant 0 \tag{4b}$$

$$\psi_{yy} = 0 = \psi_x = \theta_y \qquad \text{at } y = 0,\ x < 0. \tag{4c}$$

In the limit of very large Rayleigh number, or small ε, the governing equations (1) and (2) reduce to the boundary-layer equations, i.e. the first-order inner problem, and these equations have been solved previously by Cheng and Minkowycz [10]. The solution for large, but finite, values of the Rayleigh numbers is not found by means of a standard perturbation analysis which assumes a power-series expansion in ε in both the inner and outer regions.

2.1. *The inner expansion*

The stream and temperature functions in the boundary layer are postulated as

$$\psi(x, y, \varepsilon) = \varepsilon[\psi_1(x, Y) + \varepsilon\psi_2(x, Y) + \text{h.o.t.}] \tag{5a}$$

$$\theta(x, y, \varepsilon) = \theta_1(x, Y) + \varepsilon\theta_2(x, Y) + \text{h.o.t.} \tag{5b}$$

where the inner variable, Y, is

$$Y = y/\varepsilon \tag{6}$$

(the possibility of eigenfunctions will be considered later). Substitution of expansions (5) into equations (1) and (2) and collecting up terms of equal order in ε will result in the perturbation equations. The first-order perturbation equations resulting from this process are

$$\varepsilon^0:\ \psi_{1YY} = \theta_{1Y} \tag{7a}$$

$$\theta_{1YY} = \theta_{1x}\psi_{1Y} - \theta_{1Y}\psi_{1x} \tag{7b}$$

with the boundary conditions

$$\psi_{1x}(x,0) = 0, \quad \theta_1(x,0) = 1 \tag{8a}$$

$$\psi_{1Y}(x,\infty) \to 0, \quad \theta_1(x,\infty) \to 0. \tag{8b}$$

We note that equations (7) are the boundary layer equations found by Cheng and Minkowycz [10] for the free convection along a semi-infinite vertical heated vertical plate embedded in a porous medium.

The second-order perturbation equations are

$$\varepsilon^1: \ \psi_{2YY} = \theta_{2Y} \tag{9a}$$

$$\theta_{2yy} = \theta_{1x}\psi_{2Y} + \theta_{2x}\psi_{1Y} - \theta_{1Y}\psi_{2x} - \theta_{2Y}\psi_{1x} \tag{9b}$$

with boundary conditions

$$\psi_{2x}(x,0) = 0 \text{ and } \theta_2(x,0) = 0 \text{ or } \theta_{2y}(x,0) = 0 \tag{10}$$

and $\psi_{2Y}(x,\infty)$ and $\theta_2(x,\infty)$ match with the outer expansions which remain to be determined.

2.2. *The outer expansion*

In the outer region the flow is irrotational and isothermal, that is $\theta \equiv 0$, and thus the following expansion for the stream function is assumed

$$\psi(x,y,\varepsilon) = \varepsilon[\tilde{\psi}_1(x,y) + \varepsilon\tilde{\psi}_2(x,y) + \text{h.o.t.}] \tag{11}$$

where $\tilde{\psi}_1$ satisfies the Laplace equation

$$\Delta^2\tilde{\psi}_1 = 0 \tag{12}$$

with the boundary condition $\tilde{\psi}_1(x,0)$ that matches with the inner expansion at the edge of the boundary layer, and the infinity condition

$$\tilde{\psi}_{1y}(x,\infty) = 0. \tag{13}$$

Solutions of equations (7)–(13) will provide a complete description of the thermal and flow fields to the order ε^2.

2.3. *The first-order inner solution*

The introduction of the similarity transformation

$$\psi_1 = x^{(\lambda+1)/2} f_1(\eta), \quad \theta_1(\eta) = x^\lambda g_1(\eta) \tag{14a,b}$$

where

$$\eta = Yx^{(\lambda-1)/2} \tag{15}$$

as suggested by Cheng and Minkowycz [10] allows equations (7) to be reduced to the set of ordinary differential equations

$$f_1' = g_1 \tag{16a}$$

$$g_1'' + \frac{(\lambda+1)}{2} f_1 g_1' - \lambda f_1' g_1 = 0 \tag{16b}$$

subject to the boundary conditions

$$f_1(0) = g_1(0) - 1 = g_1(\infty) = 0. \tag{17}$$

In the above $\lambda = 0$ corresponds to the constant plate temperature problem whilst $\lambda = 1/3$ corresponds to the constant heat flux at the plate. Here primes denote derivatives with respect to η. The problem for (f_1, g_1) has been previously solved numerically by Cheng and Hsu [2]. These calculations have again been performed and we find that

constant plate temperature

$$a_1 = f_1(\infty) = 1.6161, \quad g_1'(0) = -0.4438; \tag{18a}$$

constant heat flux at the plate

$$b_1 = f_1(\infty) = 1.3016, \quad g_1'(0) = -0.6776. \tag{18b}$$

2.4. *The first-order outer solution*

In the outer region the first-order stream function, $\tilde{\psi}_1$, is governed by equation (12) with the matching condition

$$\tilde{\psi}_1(x,0) = \begin{cases} f_1(\infty)x^{(\lambda+1)/2}, & x \geqslant 0 \quad (19a) \\ 0, & -1 < x < 0 \quad (19b) \end{cases}$$

and the infinity condition (13). An additional requirement is that the bounding horizontal wall is a streamline, as no boundary layer is formed on this wall, and it may be expressed as

$$\tilde{\psi}_1(x = 1, y) = 0, \quad |y| \geqslant 0. \tag{20}$$

An analytical solution to equation (12) subject to boundary conditions (13), (19) and (20) can be obtained, for both the cases under consideration here, of the form

$$\tilde{\psi}_1 \sin\left(\frac{\lambda+1}{2}\pi\right) = f_1(\infty)\left\{(x^2+y^2)^{(\lambda+1)/4} \times \sin\left[\frac{(\lambda+1)}{2}\left(\pi - \tan^{-1}\frac{y}{x}\right)\right] - [(x+2)^2+y^2]^{(\lambda+1)/4} \times \sin\left[\frac{(\lambda+1)}{2}\tan^{-1}\frac{y}{(x+2)}\right]\right\}. \tag{21}$$

From now on we will consider separately the cases of the isothermal plate and the uniform flux plate, respectively.

(i) *Isothermal plate*. The matching of the inner and the outer expansions requires that

$$\psi_{2Y}(x,\infty) = -\frac{1}{2}a_1 x^{-1/2}\left(\frac{x}{x+2}\right)^{1/2} \tag{22a}$$

$$\theta_2(x,\infty) = 0 \tag{22b}$$

which suggests a solution of equations (9) in the inner region, of the form

$$\psi_2 = F(x,\eta), \quad \theta_2 = x^{-1/2}G(x,\eta). \tag{23a,b}$$

Thus the governing equations become

$$F'' = G' \tag{24a}$$

$$G'' + \frac{1}{2}(f_1 G)' = x\left(f_1'\frac{\partial G}{\partial x} - g_1'\frac{\partial F}{\partial x}\right) \tag{24b}$$

which have to be solved subject to the boundary conditions

$$F(x,0) = G(x,0) = 0 \tag{25a}$$

$$F'(x,\infty) = -\frac{1}{2}a_1\left(\frac{x}{x+2}\right)^{1/2}, \quad G(x,\infty) = 0. \tag{25b,c}$$

We see that equations (24) are non-similar and therefore their solution is obtained in terms of two coordinate expansions which are valid for small and large values of x, respectively.

(a) *Small values of* x. For $x \ll 1$, the formal expansion of $F'(x,\infty)$ is

$$F'(x,\infty) = \sum_{n=0}^{\infty} A_n x^{n+1/2} \tag{26}$$

where

$$A_n = -\frac{a_1}{2^{n+3/2}}\frac{\Gamma(1/2)}{\Gamma(1+n)\Gamma(1/2-n)} \tag{27}$$

and Γ is the Gamma function. Guided by the expansion (26), we write

$$F = \sum_{n=0}^{\infty} F_n(\eta)x^{n+1/2} \tag{28a}$$

$$G = \sum_{n=0}^{\infty} G_n(\eta)x^{n+1/2} \tag{28b}$$

so that the system of partial differential equations (24) can be reduced to the system of ordinary differential equations

$$F_n'' = G_n' \tag{29a}$$

$$G_n'' + \tfrac{1}{2}f_1G_n' - nf_1'G_n + (\tfrac{1}{2}+n)g_1'F_n = 0. \tag{29b}$$

Boundary conditions (25) become, on using expressions (26) and (27)

$$F_n(0) = G_n(0) = 0 \tag{30a}$$

$$F_n'(\infty) = A_n, \quad G_n(0) = 0. \tag{30b,c}$$

(b) *Large values of* x. For $x \gg 1$, the asymptotic form of $F'(x,\infty)$ may be expressed in inverse powers of x as

$$F'(x,\infty) = -\tfrac{1}{2}a_1 + O(1/x) \tag{31}$$

and therefore the asymptotic forms for F and G are

$$F = \tilde{F}(\eta) + O(1/x) \tag{32a}$$

$$G = \tilde{G}(\eta) + O(1/x) \tag{32b}$$

as $x \to \infty$. On substitution of expressions (32) into equations (24) and equating order one terms gives

$$\tilde{F}'' = \tilde{G}' \tag{33a}$$

$$\tilde{G}'' + \tfrac{1}{2}(f_1\tilde{G})' = 0 \tag{33b}$$

with the boundary conditions

$$\tilde{F}(0) = \tilde{G}(0) = 0 \tag{34a}$$

$$\tilde{F}'(\infty) = -\tfrac{1}{2}a_1, \quad \tilde{G}(\infty) = 0. \tag{34b,c}$$

It is observed that equations (33) and boundary conditions (34) are identical to those derived by Daniels and Simkins [4] in their study of free convection flow in a corner which is embedded in a porous medium. The solution of equations (33) subject to the boundary condition (34) is

$$\tilde{F} + -\tfrac{1}{2}a_1\eta, \quad \tilde{G} \equiv 0. \tag{35}$$

(c) *Moderate values of* x. A standard Crank–Nicolson type solver is used to solve the parabolic partial differential equations (24) subject to boundary conditions (25). The method starts with the solution which is valid for small values of x and marches in x until the asymptotic solution for large x is obtained.

(ii) *Uniform flux plate*. The solution procedure for this problem, in essence, parallels that of the preceding case. Now, from equation (21), we obtain

$$\tilde{\psi}_{1y}(x,0) = \frac{2}{3}x^{-1/3}b_1\cot\left(\frac{\pi}{3}\right)\left[1-2\left(\frac{x}{x+2}\right)^{1/3}\right]. \tag{36}$$

It is observed, from expression (36), that the displacement speed becomes zero at $x = x_c$ where x_c is given by

$$x_c = 2/7. \tag{37}$$

Therefore, the nature of the flow at the edge of the boundary layer changes depending upon the value of x. For $x < 2/7$ the displacement speed is positive; at $x = 2/7$, the displacement speed is zero and the first-order contribution vanishes; for $x > 2/7$, the displacement speed is negative.

As before, the second-order boundary layer equations (9) in terms of the variable

$$\psi_2 = \hat{F}(x,\eta), \quad \theta_2 = x^{-1/3}\hat{G}(x,\eta) \tag{38a,b}$$

reduce to equations similar to equations (24) and are therefore not presented here. However, the functions $\hat{F}$ and $\hat{G}$ take the following forms:

for small values of x

$$\hat{F} = f_2(\eta) + \sum_{n=0}^{\infty} F_n(\eta)x^{n+1/3} \tag{39a}$$

$$\hat{G} = g_2(\eta) + \sum_{n=0}^{\infty} G_n(\eta)x^{n+1/3} \tag{39b}$$

where $\hat{F}'(x,\infty)$ behaves as

$$\hat{F}'(x,\infty) = B_1 + \sum_{n=0}^{\infty} \hat{A}_n x^{n+1/3} \tag{40}$$

with

$$B_1 = \frac{2}{3}b_2\cot\left(\frac{\pi}{3}\right) \tag{41a}$$

$$\hat{A}_n = -\frac{2}{3}b_1\operatorname{cosec}\left(\frac{\pi}{3}\right)\frac{1}{2^{n+1/3}}\frac{\Gamma(1/3)}{\Gamma(1+n)\Gamma(1/3-n)}; \tag{41b}$$

for large values of x

$$\hat{F} = \hat{\hat{F}}(\eta) + O(1/x) \quad (42a)$$

$$\hat{G} = \hat{\hat{G}}(\eta) + O(1/x) \quad (42b)$$

where

$$\hat{F}'(x, \infty) = -B_1 + O(1/x). \quad (43)$$

Equations for F and G are identical to the equations for f_2 and g_2 which have to be solved for the same boundary conditions except for a different sign in conditions at $\eta = \infty$. Thus it is concluded that

$$f_2'(0) = -\hat{\hat{F}}(0) = 0.1677, \quad g_2(0) = -\hat{\hat{G}} = -0.3333. \quad (44)$$

For moderate values of x a numerical procedure, similar to that described for the isothermal plate problem, is again employed.

In order to complete the solution of this problem we must search for the existence of the eigenfunctions which identically satisfy the boundary conditions at $\eta = 0$ and ∞. But, as in the problem of free convection along a single-semi-infinite vertical plate which is embedded in a porous medium, the eigensolutions associated with equations (5) introduce a term which lies between the second- and third-order approximation in each of the series. It is therefore concluded that the boundary layer expansions (5) are appropriate up to order ε^2.

3. RESULTS AND DISCUSSION

The primary importance in this problem is the fluid flow pattern and the variation of the Nusselt number with the distance along the plate. A set of streamlines, deduced from expression (21), corresponding to both the isothermal plate and the uniform flux plate are shown in Figs. 2 and 3, respectively. The effect of the solid horizontal boundary at $x = -1$ is to prevent fluid being entrained into the boundary layer from the region $x < -1$ and although schematically the streamlines are similar in the two problems they are very different in detail.

In the case of the uniform flux plate it is observed from Fig. 3 that ψ_{1y}, at $y = 0$, $x > 0$, is negative near $x = 0$ but becomes positive as x increases. The transition from negative to positive values occurring at $x = x_c = 2/7$, as predicted by equation (37). In the isothermal plate case Fig. 2 shows that ψ_{1y}, at $y = 0$, $x > 0$, is always positive as indicated by equation (22a). Further, since the flux of fluid entering the boundary layer is larger for the uniform flux plate than the isothermal plate then the strength of the flow outside the boundary layer must be greater. This is confirmed by an examination of expression (21). Therefore the fluid, for the uniform flux plate, has to enter the boundary layer much more rapidly than for the isothermal plate problem. This results in the streamlines being more horizontal for the uniform flux plate than for the isothermal plate. This result is confirmed by the streamline patterns in Figs. 2 and 3.

The local Nusselt number for small values of x can be expressed as

$$\text{for (i):} \quad Nu_x/Ra_x^{1/2} = 0.4438 - Ra_x^{-1/2} \sum_{n=0}^{\infty} G_n'(0)x^{n+1/2} + O(Ra_x^{-1}) \quad (45a)$$

$$\text{for (ii):} \quad Nu_x/(Nu_x)_{\text{B.L.}} = 1 + \frac{Ra_x^{-1/2}}{g_1'(0)} \times \left[g_2(0) + \sum_{n=0}^{\infty} G_n(0)x^{n+1/3} \right] + O(Ra_x^{-1}) \quad (45b)$$

where $g_1'(0) = -0.6776$ and $g_2(0) = -0.3333$. In the above $(Nu_x)_{\text{B.L.}}$ denotes the local Nusselt number for the first-order boundary layer solution. The results to $n = n_0 = 14$ terms for $G_n'(0)$ and $\hat{G}_n(0)$ are given in Table 1.

For large values of x, the results for the leading terms in the expansions are

$$\text{for (i):} \quad Nu_x/Ra_x^{1/2} = 0.4438 + O(Ra_x^{-1}) \quad (46a)$$

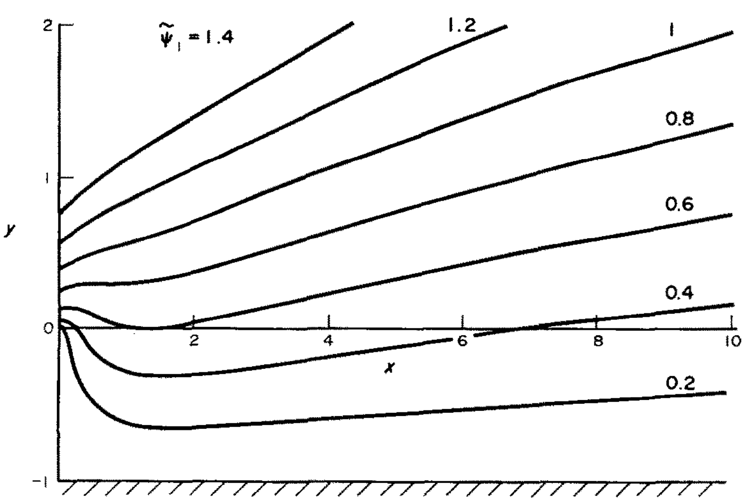

FIG. 2. The streamlines associated with the outer flow for an isothermal plate.

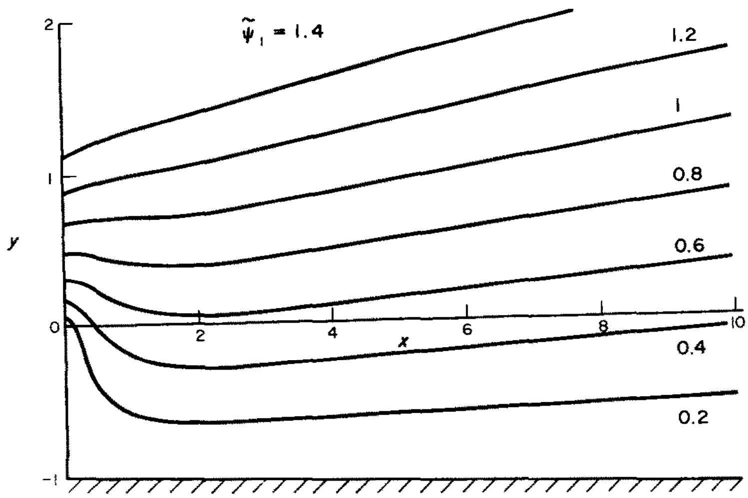

FIG. 3. The streamlines associated with the outer flow for a uniform flux plate.

Table 1. Second-order solution for small x

n	A_n	$G_n'(0)$	A_n	$G_n(0)$
0	-5.7139×10^{-1}	2.1477×10^{-1}	-7.9528×10^{-1}	4.7933×10^{-1}
1	1.4285×10^{-1}	-7.2668×10^{-2}	1.3253×10^{-1}	-6.3253×10^{-2}
2	-5.3567×10^{-2}	2.8271×10^{-2}	-4.4182×10^{-2}	1.7741×10^{-2}
3	2.2320×10^{-2}	-1.1785×10^{-2}	1.7182×10^{-2}	-6.0274×10^{-3}
4	-9.7649×10^{-3}	5.1174×10^{-3}	-7.1592×10^{-3}	2.2498×10^{-3}
5	4.3942×10^{-3}	-2.2825×10^{-3}	3.1023×10^{-3}	-8.8907×10^{-4}
6	-2.0140×10^{-3}	1.0373×10^{-3}	-1.3788×10^{-3}	3.6512×10^{-4}
7	9.3508×10^{-4}	-4.7789×10^{-4}	6.2375×10^{-4}	-1.5417×10^{-4}
8	-4.3832×10^{-4}	2.2248×10^{-4}	-2.8588×10^{-4}	6.6477×10^{-5}
9	2.0698×10^{-4}	-1.0443×10^{-4}	1.3235×10^{-4}	-2.9139×10^{-5}
10	-9.8137×10^{-5}	4.9338×10^{-5}	-6.1765×10^{-5}	1.2942×10^{-5}
11	4.6924×10^{-5}	-2.3435×10^{-5}	2.9011×10^{-5}	-5.8105×10^{-6}
12	-2.2484×10^{-5}	1.1182×10^{-5}	-1.3700×10^{-5}	2.6324×10^{-6}
13	1.0810×10^{-5}	-5.3551×10^{-6}	6.4985×10^{-6}	-1.2017×10^{-6}
14	-5.2119×10^{-6}	2.5729×10^{-6}	-3.0945×10^{-6}	5.5219×10^{-7}

for (ii): $Nu_x/(Nu_x)_{\text{B.L.}}$

$$= 1+\frac{\hat{\hat{G}}(0)}{g_1'(0)}Ra_x^{-1/2}+O(Ra_x^{-1})$$

$$= 1-0.4919Ra_x^{-1/2}+O(Ra_x^{-1}). \quad (46b)$$

Figure 4 shows the variation of the second-order boundary layer correction to the local Nusselt number, $(Nu_x/Ra_x^{1/2}-0.4438)Ra_x^{1/2}$, as a function of the distance along the plate, x, for the isothermal plate problem. For large values of x the complete numerical solution tends to the asymptotic value of zero as predicted by equation (46a). At small values of x the effects of taking an increasing number of terms in expansion (45a) is shown. Clearly we do not expect the asymptotic solution for small values of x to be valid for $x > 2$ because of the use of expansion (26). The full numerical solution gives excellent agreement with the series solution (45a) for $x \lesssim 1.5$.

Figure 5 shows the variation of the second-order boundary layer correction to the local Nusselt number, $(Nu_x/(Nu_x)_{\text{B.L.}}-1)Ra_x^{1/2}$, as a function of the distance along the plate for the uniform flux plate. As in the case of isothermal plate problem it is observed that the complete numerical solution matches the asymptotic solutions (45b) and (46b). Similar observations as were made above for the isothermal plate also apply in this case.

It is concluded that the first-order boundary layer solution always overestimates the Nusselt number in the case of the isothermal plate. However, for the uniform flux plate the first-order boundary layer solution underestimates the Nusselt number for $x \lesssim 0.4$ and overestimates it for $x \gtrsim 0.4$.

Higher-order corrections to the boundary layer solutions can easily be obtained using the method of

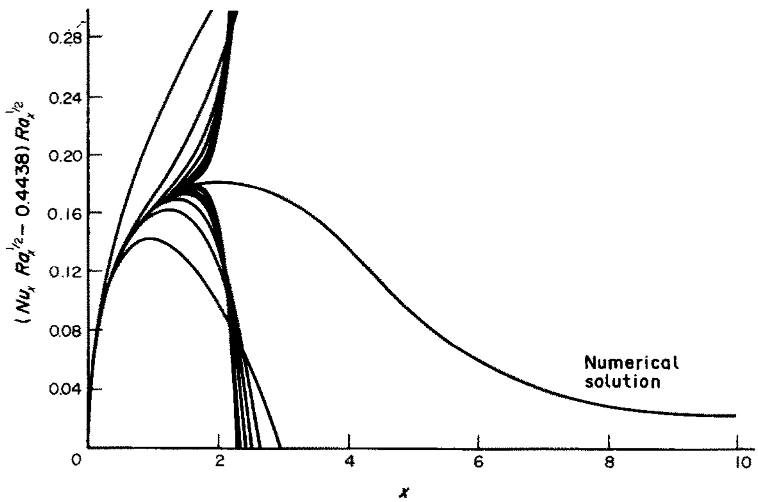

FIG. 4. The variation of $(Nu_x Ra_x^{1/2} - 0.4438) Ra_x^{1/2}$ as a function of x for the isothermal plate. The number of terms used in series (45a) being 0, 14, 12, 10, 8, 6, 4 and 2 for curves going to plus infinity, reading from left to right and being 13, 11, 9, 7, 5, 3 and 1 for curves going to minus infinity, again reading from left to right.

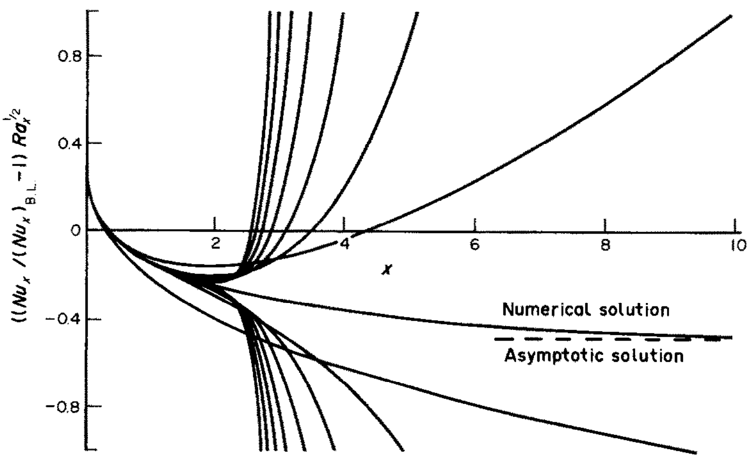

FIG. 5. The variation of $((Nu_x/Nu_x)_{B.L.} - 1)Ra_x$ as a function of x for the uniform flux plates. The number of terms used in series (45b) being 13, 11, 9, 7, 5, 3 and 1 for curves going to plus infinity, reading from left to right, and being 14, 12, 10, 8, 6, 4, 2 and 0 for curves going to minus infinity, reading from left to right.

matched asymptotics as described above. However, in the non-porous media situation this is not possible as a viscous boundary layer forms on the horizontal surface and this must also be analysed. Also due to the formation of the boundary layer a jet will form near $y = 0$, $-1 < x < 0$ which will then interact with the higher-order corrections in the boundary layer on $y = 0$, $x > 0$. In the porous media problem no viscous or thermal boundary layers exist on the horizontal boundary at $x = -1$ and this boundary is only playing a passive role in that it only serves as a boundary condition for the outer inviscid flow.

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CONVECTION NATURELLE SUR UNE SURFACE VERTICALE SEMI-INFINIE LIMITEE PAR UNE PAROI HORIZONTALE DANS UN MILIEU POREUX

Résumé—On étudie la convection naturelle permanente le long d'une plaque plane, verticale semi-infinie qui est placée dans un milieu poreux à une distance arbitraire d au-dessus d'une paroi horizontale. Les équations de couche limite de premier et de second ordre et les équations de l'écoulement non visqueux externe sont étudiées pour trouver les effets des grands, mais finis, nombres de Rayleigh. Des résultats sont obtenus pour les deux cas dans lesquels la plaque plane est (1) isotherme et (2) à flux uniforme. On trouve que les solutions de couche limite de premier ordre surestiment le nombre de Nusselt local excepté pour des distances le long de la plaque, à flux uniforme, inférieures à 0,4d environ.

FREIE KONVEKTION AN EINER HALBUNENDLICHEN SENKRECHTEN OBERFLÄCHE MIT HORIZONTALER BEGRENZUNG IN EINEM PORÖSEN MEDIUM

Zusammenfassung—Die stationäre freie Konvektionsströmung entlang einer halbunendlichen senkrechten ebenen Platte in beliebiger Entfernung d über einer waagerechten Fläche in einem gesättigten porösen Medium wird betrachtet. Die Grenzschichtgleichungen erster und zweiter Ordnung und die Gleichungen für den äußeren Bereich, der reibungsfrei betrachtet wird, werden untersucht, um den Eingfluß von sehr großen, endlichen Rayleigh-Zahlen herauszufinden. Ergebnisse wurden für den Fall einer isothermen ebenen Platte und für konstante Wärmestromdichte berechnet. Die Lösungen der Grenzschichtgleichung erster Ordnung liefern zu hohe Werte für die Nusselt-Zahl, außer für den Fall der einheitlichen Wärmestromdichte bei Abständen von der Plattenunterkante von weniger als ungefähr 0,4d.

ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ В ПОРИСТОЙ СРЕДЕ ОТ ПОЛУБЕСКОНЕЧНОЙ ВЕРТИКАЛЬНОЙ ПОВЕРХНОСТИ, ОГРАНИЧЕННОЙ ГОРИЗОНТАЛЬНОЙ СТЕНКОЙ

Аннотация—Проведено исследование стационарной естественной конвекции вблизи полубесконечной вертикальной плоской пластины, которая помещена в насыщенную пористую среду, и на произвольном расстоянии d ограничена горизонтальной стенкой. Для определения влияния больших, но конечных значений чисел Рэлея уравнения пограничного слоя первого и второго порядка решаются совместно с уравнениями для внешнего течения невязкой жидкости. Результаты получены для двух случаев, когда плоская пластина (i) изотермична и (ii) на ней поддерживается однородный тепловой поток. Найдено, что решения уравнений пограничного слоя первого порядка дают завышенные значения местного числа Нуссельта, кроме расстояний, меньших приблизительно 0,4d, для пластины с однородным тепловым потоком.